

# Phase collapse and revival of a Bose-Einstein condensate induced by an off-resonant optical probe field

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We consider a single mode atomic Bose-Einstein condensate interacting with an off-resonant optical probe field. We show that the condensate phase revival time is dependent on the atom-light interaction, allowing optical control on the atomic collapse and revival dynamics through frequencies detuning. Also, incoherent effects over the condensate phase are included by considering a continuous photo-detection over the probe field. We consider conditioned and unconditioned photo-counting events and verify that no extra control upon the condensate is achieved by the probe photo-detection. Whether or not conditioned to the number of photons detected, the photo-detection process contributes to suppresses the typical collapse and revival dynamics.

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## I. INTRODUCTION

Collapse and revival phenomena in Bose-Einstein condensates (BECs) have been investigated since 1996 [1], shortly after its experimental achievement with ultracold atoms [2, 3]. They are a consequence of the quantized structure of the matter field and the coherent interactions between the atoms through atomic collisions (See [4] and references therein). Dynamically originated due the presence of nonlinearities, phase collapse and revival is similar in nature to the collapse and revival appearing in Rabi oscillations when a single two-level atom interacts with a single mode of a quantized optical field, or when a coherent light field propagates in a nonlinear medium [5]. The possibility of coexistence of nonlinear effects due to external influences opens an interesting venue for investigating and controlling [6] such phenomena in BECs. Particularly, the control of phase collapse in BECs is extremely relevant for atomic interferometry since it might prevent phase diffusion, while allowing a performance below the standard quantum limit [7]. Several attempts on both developing new tools and pushing forward the possible frontiers for collapse control have been investigated for double-well condensates (See [6] and references therein). A possible approach is to consider the action of an external light probe over the atomic system, although, in general this leads to an even worse situation where the light field itself induces phase collapse in an irreversible manner. It is expected that, unless a time dependent interaction is employed, no further control is achieved.

Nonetheless, the interaction of the atomic system with a quantized light probe field can be engineered with the assistance of an additional pump field (See [8] for an excellent review on this topic). This scheme allows, e.g., the simultaneous amplification of atomic and optical fields, as well as control

of the atomic field statistical properties [9–11]. Previously, with a setup relying on the atomic system-probe field interaction mediated through a classical pump [12], we showed that under continuous photo-counting, the moments of the probe light photon number might carry information about the even moments of the atom number. However, neither the possibility of controlling of atomic properties with the proposed setup, or a detailed analyzes about the effects of conditioned single photo-counting events over the BEC state, was carried out. Such analyzes is important, given the interest in phase collapse time, since in some situations the detection process allows an additional control or better inference of parameters. For example, in [13] an optical probe continuous detection allows one to create relative phase of two spatially separated atomic BEC. Also, as showed in [14], even dissipative environments can be useful to tailor dynamics of states and phase in cold atomic samples.

To address these questions, in this paper we analyze the situation in which a single mode atomic BEC is coupled to a quantized optical probe field, through an undepleted optical pump. The nonlinear nature of the engineered interaction between the atoms and the probe field, induces a proper dynamics of collapse and revival of the atomic phase, even in the absence of atomic collisions for a very diluted atomic gas. We show that the characteristic revival time, depends on the commensurability between the parameters such as the coupling constants and the detuning between the transition frequency of the two level atoms (that represents our BEC) and the frequency of the pump field, as well as, the atomic collision parameter. This allows control over the atomic collapse and revival through the effective atom-light coupling parameters. In addition, we include continuous photo-counting over the probe field and analyze its possible control over the collapse and revival times. We conclude that no further control is achieved by these measurements.

The paper is organized as follows: In Section II we deduce the model for a BEC trapped in a single well potential, interacting with a classical undepleted optical field and a quantized optical probe mode in the far-off resonant regime. In Section III, we analyze the collapse and revival dynamics of

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the BEC phase using the Husimi function and the variance of the phase operator. We show how the coupling parameters between atoms and light determine and enable to control the collapse and revival dynamics. In Section IV we include an incoherent process through a continuous photo-detection on the quantized optical probe field. The effect of optical photo-counting over the BEC phase dynamics is analyzed. Finally, in Section V we present the conclusions.

## II. MODEL

We consider a system of bosonic two-level atoms interacting via two-body collisions and coupled through electric-dipole interaction with two single-mode running wave optical fields of frequencies  $\omega_1$  and  $\omega_2$ , and wavevectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$ , respectively. The probe optical field (1) is treated quantum mechanically, while the pump (2) is undepleted and is treated classically. Both fields are assumed to be far off-resonance from any electronic transition, and the excited state population is small so that spontaneous emission may be neglected. Similarly, collisions among excited state atoms, as well as, collisions between ground state atoms with excited state ones, are very improbable and can also be neglected. In this regime the excited state can be adiabatically eliminated and the ground state atomic field plus the optical probe evolve coherently under the effective Hamiltonian [10, 12, 15]

$$H = \int d^3\mathbf{r} \Psi^\dagger(\mathbf{r}) \left[ H_0 + \frac{U}{2} \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}) + \hbar \frac{|g_2 \alpha_2|^2}{\Delta} \right] \Psi(\mathbf{r}) \\ + \hbar \int d^3\mathbf{r} \Psi^\dagger(\mathbf{r}) \left[ \frac{g_1^* g_2 \alpha_2}{\Delta} a_1^\dagger e^{-i\mathbf{k} \cdot \mathbf{r}} + \text{H.c.} \right] \Psi(\mathbf{r}) \\ + \hbar \left[ (\omega_1 - \omega_2) + \frac{|g_1|^2}{\Delta} \int d^3\mathbf{r} \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}) \right] a_1^\dagger a_1. \quad (1)$$

$\Psi(\mathbf{r})$  is the ground state atomic field operator which satisfies the usual bosonic commutation relations  $[\Psi(\mathbf{r}), \Psi^\dagger(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}')$ .  $H_0 = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r})$  is the trapped atoms Hamiltonian, where  $m$  is the atomic mass and  $V(\mathbf{r})$  is the trap potential.  $\Delta = \omega_2 - \nu$  is the detuning between the atomic transition and the optical pump frequencies,  $g_1$  and  $g_2$  are the atom-light coupling coefficients, and  $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ . The operators  $a_1$  and  $a_1^\dagger$  (given in a rotating frame with frequency  $\omega_2$ ) satisfy the commutation relation  $[a_1, a_1^\dagger] = 1$ , and  $\alpha_2$  is the pump amplitude. Two-body collisions were included in the s-wave scattering limit, where  $U = \frac{4\pi\hbar^2 a}{m}$  and  $a$  is the s-wave scattering length [16].

We expand the atomic field operators in terms of the orthogonal set of trap eigenmodes  $\{\varphi_n(\mathbf{r})\}$ , as

$$\Psi(\mathbf{r}) = \sum_n c_n \varphi_n(\mathbf{r}), \quad (2)$$

where the trap eigen-modes satisfies the orthonormality relation  $\int d^3\mathbf{r} \varphi_m^*(\mathbf{r}) \varphi_n(\mathbf{r}) = \delta_{mn}$  and the eigenvalue equation  $H_0 \varphi_n(\mathbf{r}) = \hbar \tilde{\omega}_n \varphi_n(\mathbf{r})$  ( $\tilde{\omega}_n$  is the corresponding trap eigen-frequencies).  $c_n$  is the atom annihilation operator in the mode

$n$ , and together with the creation operator, satisfies the regular commutation relation:  $[c_n, c_n^\dagger] = \delta_{nn}$ . With the expansion (2), the Hamiltonian (1) takes the following form

$$H = \hbar \sum_n \left( \tilde{\omega}_n + \frac{|g_2 \alpha_2|^2}{\Delta} \right) c_n^\dagger c_n + \hbar \sum_{ijlm} \kappa_{ijlm} c_i^\dagger c_j^\dagger c_l c_m \\ + \hbar \sum_{mn} \left( \frac{g_1^* g_2 \alpha_2}{\Delta} \chi_{mn} a_1^\dagger c_n^\dagger c_m + \frac{g_2^* g_1 \alpha_2^*}{\Delta} \chi_{nm} a_1 c_m^\dagger c_n \right) \\ + \hbar \left( \delta + \frac{|g_1|^2}{\Delta} \sum_n c_n^\dagger c_n \right) a_1^\dagger a_1, \quad (3)$$

where

$$\kappa_{ijlm} = \frac{U}{2\hbar} \int d^3\mathbf{r} \varphi_i^*(\mathbf{r}) \varphi_j^*(\mathbf{r}) \varphi_l(\mathbf{r}) \varphi_m(\mathbf{r}), \quad (4)$$

is the inter-modes collision parameter and

$$\chi_{nm} = \int d^3\mathbf{r} \varphi_m^*(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}} \varphi_n(\mathbf{r}), \quad (5)$$

is the optical transition matrix elements from the  $m$ th state to  $n$ th state. Hamiltonian (3), accounts for the process of atomic and optical parametric amplification due momentum exchange between atoms and optical field [10]. Here, we assume a sufficiently low temperature so that all atoms form a pure BEC in the trap ground state, and to avoid scattering process which transfers atoms to other trap modes via momentum exchange we consider that  $\mathbf{k}_1 \approx \mathbf{k}_2$  (and  $\omega_1 \approx \omega_2$ ). With these assumptions, only the atomic operators  $c_0^\dagger$  ( $c_0$ ) corresponding to creation (annihilation) of atoms at the trap ground state remains, and the Hamiltonian (3) simplifies to

$$H = \hbar \left( \tilde{\omega}_0 + \frac{|\tilde{g}_2|^2}{\Delta} \right) c_0^\dagger c_0 + \hbar \kappa c_0^\dagger c_0^\dagger c_0 c_0 \\ + \hbar c_0^\dagger c_0 \left( \frac{g_1 \tilde{g}_2^*}{\Delta} a + \frac{\tilde{g}_2 g_1^*}{\Delta} a^\dagger \right) + \hbar \frac{|g_1|^2}{\Delta} c_0^\dagger c_0 a^\dagger a, \quad (6)$$

where  $\tilde{g}_2 = g_2 \alpha_2$  and  $\kappa = \frac{U}{2\hbar} \int d^3\mathbf{r} |\varphi_0(\mathbf{r})|^4$  is the collision parameter between the atoms in the trap ground state. The optical probe mode index was dropped in order to simplify the notation.

Assuming that initially the atoms plus optical field joint state is completely disentangled, i.e.,  $|\psi(0)\rangle = |\alpha\rangle \otimes |\beta\rangle$ , where both, atomic ( $|\alpha\rangle$ ) and optical field ( $|\beta\rangle$ ) are supposed to be coherent states, the time evolution of the combined system can be obtained exactly from the Hamiltonian (6) (See the appendix for a detailed derivation), and is given by

$$|\psi(t)\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_m \frac{\alpha^m e^{-\Phi_m(t)}}{\sqrt{m!}} |m\rangle \otimes |\beta_m(t)\rangle, \quad (7)$$

where

$$\beta_m(t) = \beta e^{-i\frac{|g_1|^2 m}{\Delta} t} + \frac{\tilde{g}_2}{g_1} (e^{-i\frac{|g_1|^2 m}{\Delta} t} - 1), \quad (8)$$

is the probe field coherent state time dependent amplitude and

$$\Phi_m(t) = \frac{1}{2} \left[ |\beta_m(t)|^2 - |\beta|^2 \right] + \frac{\tilde{g}_2}{g_1} [\beta_m(t) - \beta] - i\tilde{\omega}_0 m t - i\kappa m(m-1)t, \quad (9)$$

is the relative phase introduced by the dynamics. The density operator for the combined system is  $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$  from which we can obtain the BEC state by tracing over the optical state,  $\rho_A(t) = \text{Tr}_L \rho(t)$ , where  $A$  and  $L$  stand for atoms and light, respectively.

### III. CONDENSATE PHASE DYNAMICS

Analyzing each term from Hamiltonian (6) separately, we are able to visualize different scenarios. For instance, its second term plays a key role on the collapse and revival of the atomic phase dynamics, typical of one-mode BECs (See Fig. 1 in Greiner et al. [4] for the atomic Husimi function  $Q_A(\alpha_A, t) = \frac{1}{\pi} \langle \alpha_A | \rho_A(t) | \alpha_A \rangle$  corresponding here to  $\frac{|g_1|^2}{\Delta} \ll \kappa$ ). After evolving into states of totally uncertain phases and also some exact superpositions of coherent states, the atomic state fully recover the initial phase at the revival time  $t_{rev}^C = \pi/\kappa$ .

The third and fourth terms in Hamiltonian (6) describe the couplings between atoms and the quantum optical probe. The third term corresponds to a transfer of photons from the classical undepleted pump optical field to the quantized probe mode mediated by the atoms. The fourth term is quite relevant since it will also contribute to the collapse and revival of the BEC phase (and of the optical probe) due to the cross-Kerr type of nonlinearity. The regime  $\frac{|g_1|^2}{\Delta} \gg \kappa$ , for a small detuning (large enough though to prevent spontaneous emission [8]) or for a very diluted atomic gas ( $\kappa \approx 0$ ), is depicted in Fig. 1. The collapse and revival dynamics occurs at a completely distinct revival time  $t_{rev}^L = 2\pi\Delta/|g_1|^2$ , depending on the atom-light interaction parameter. Similarly the probe light field will also show a collapse and revival dynamics. The only extra effect is due to the third term in Hamiltonian (6), which displaces an initial light field coherent state depending on the number of atoms in the BEC.

A nontrivial dynamics occurs when both the second and fourth terms in (6) are of the same order, which can be reached by varying the detuning  $\Delta$ . In a general way the revival occurs whenever the terms in a expansion spanned by Fock states of the atomic state are in phase. While it is easy to describe this when only the collision term or the interaction with the optical probe are on, the situation is more complicated when both terms are relevant. To show that, let us analyze the behavior of the variance of the phase operator for the atomic mode [17]. For an initial coherent state of a large amplitude  $\alpha$ , the phase variance is approximately  $V(\phi) = 1/4|\alpha|^2$  and clearly shows that the phase is well defined for large  $\alpha$ . For the chosen amplitude of  $|\alpha|^2 = 3$  the phase revival, as depicted in Fig. 2, occurs every time  $V(\phi)$  approaches zero. Note that the collapse and revival dynamics similar to the one in Fig. 1 in Greiner et al. [4] repeats

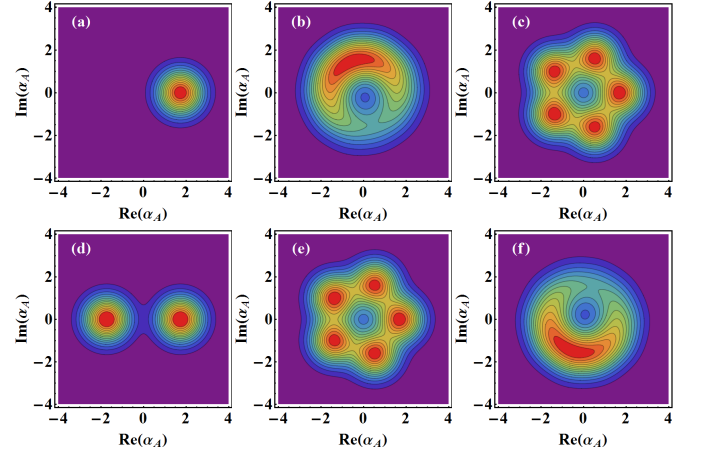


FIG. 1. Husimi function dynamics of the pre-selected atomic density operator in the limit  $\frac{|g_1|^2}{\Delta} \gg \kappa$ . In (a)  $t = 0$ ; (b)  $t = 0.1(2\pi\Delta/|g_1|^2)$ ; (c)  $t = 0.4(2\pi\Delta/|g_1|^2)$ ; (d)  $t = 0.5(2\pi\Delta/|g_1|^2)$ ; (e)  $t = 0.6(2\pi\Delta/|g_1|^2)$ ; (f)  $t = 0.9(2\pi\Delta/|g_1|^2)$ . A full revival of the initial coherent state in (a) is reached at  $t_{rev}^L = 2\pi\Delta/|g_1|^2$ .

several times in Fig. 2(a) for the atomic mode at the chosen time scale. However this behavior is very fragile and changes completely with the inclusion of a very small perturbation in  $|g_1|^2/\kappa\Delta$ . The revival time for this scenario can be investigated by expanding the collision and interaction terms in a Fock basis,  $\{|n\rangle, |m\rangle\}$  for the atoms and light field, respectively. The revival time is not dependent on the third term of the Hamiltonian (6), whose effect is only to displace an initial light field coherent state depending on the number of atoms in the BEC. Then, considering a rotating frame with frequency  $\tilde{\omega}_0 + \frac{|\tilde{g}_2|^2}{\Delta}$ , the revival time coincides with the recurrence of the initial phase, which will take place whenever the following relation is satisfied

$$e^{-i \left[ \kappa n(n-1) + \frac{|g_1|^2}{\Delta} nm \right] t} = e^{-2il\pi}, \quad (10)$$

where  $l$ ,  $m$ , and  $n$  are positive integers. This occurs for times such that

$$t_{rev} = \frac{2\kappa\Delta}{|g_1|^2} \left[ \frac{l}{\frac{\kappa\Delta}{|g_1|^2} n(n-1) + nm} \right] t_{rev}^C, \quad (11)$$

since  $t_{rev}^C = \pi/\kappa$ . The revival depends on the commensurability between  $|g_1|^2/\Delta$  and  $\kappa$ . Since  $n$  and  $m$  are integers, when  $|g_1|^2/\kappa\Delta$  is rational ( $\equiv \frac{p}{q}$ , with  $p, q$  integers) we have

$$t_{rev} = \frac{2\kappa\Delta}{|g_1|^2} \left[ \frac{lq}{pn(n-1) + qnm} \right] t_{rev}^C. \quad (12)$$

Given that  $pn(n-1) + qnm$  is an integer and  $l$  is arbitrary, there always exists a  $lq = pn(n-1) + qnm$  in the numerator of Eq. (12), so that the revival time reduces to  $t_{rev} = 2\frac{\kappa\Delta}{|g_1|^2} t_{rev}^C$ . However, for an irrational  $|g_1|^2/\kappa\Delta$ , there is no revival at all.

Let us exemplify with the inclusion of perturbations in the interaction with the light field (by decreasing the detuning). In

Fig. 2(b), for  $|g_1|^2/\kappa\Delta = 1/50$ , we see an inhibition of the number of revivals at this time scale, even though an actual revival occurs at a different time scale at  $100 t_{rev}^C$ . In a similar way Fig. 2(c) for  $|g_1|^2/\kappa\Delta = 1/5$ , shows that the revival time is  $10 t_{rev}^C$ , and in Figs. 2(d) and 2(f), for  $|g_1|^2/\kappa\Delta = 1/2$ , and  $|g_1|^2/\kappa\Delta = 1$ , where the revival time is  $4 t_{rev}^C$  and  $2 t_{rev}^C$ , respectively. However, for  $\frac{g^2}{\kappa\Delta} = \frac{2}{\pi}$ , as in Fig. 2(e), there is no phase revival, as we expected. The relevant aspect on

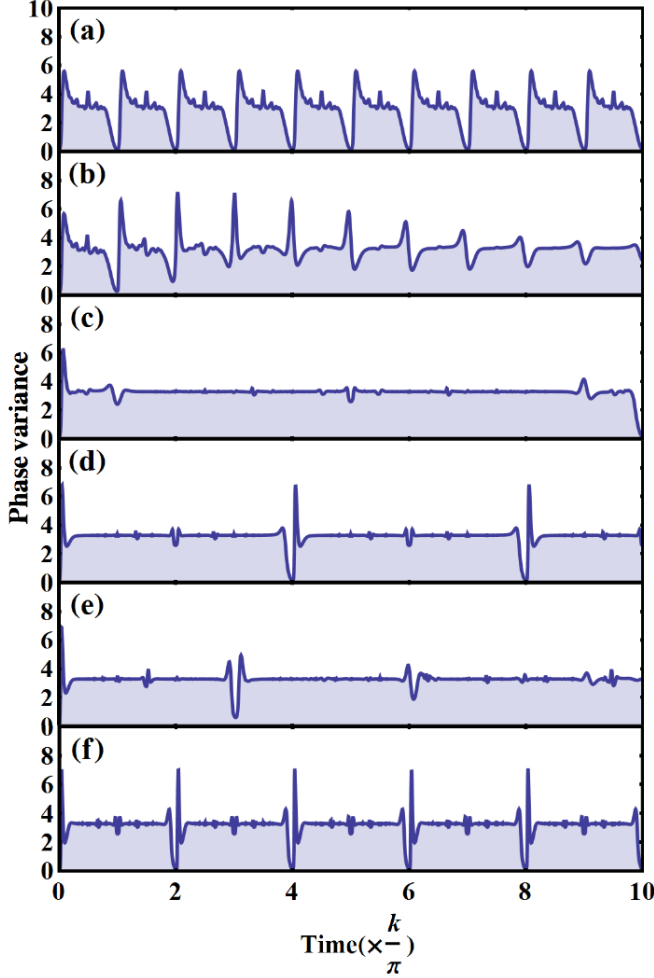


FIG. 2. Phase variance for the BEC owing to both atom collisions and interaction with the optical probe. The latter one disturbs enormously the revival time, when the atomic state returns approximately to a coherent state. From top to bottom, (a)  $|g_1|^2/\kappa\Delta = 0$ , (b)  $|g_1|^2/\kappa\Delta = 1/50$ , (c)  $|g_1|^2/\kappa\Delta = 1/5$ , (d)  $|g_1|^2/\kappa\Delta = 1/2$ , (e)  $|g_1|^2/\kappa\Delta = 2/\pi$ , and (f)  $|g_1|^2/\kappa\Delta = 1$ .

the atomic revival time change is the possibility to control it through the variation of  $\Delta$ . Whenever the revival occurs, and only then, both optical pump and the BEC are left disentangled. Besides the control over the revival, one could be interested in this available entanglement. Several entangled coherent states occur, being a typical example at exactly half of the revival time - the optical probe and BEC are approximately left in a state

$$|\psi\rangle \approx |\beta\rangle|\alpha_+\rangle + |\beta\rangle|\alpha_-\rangle, \quad (13)$$

where  $|\alpha_\pm\rangle$  are odd and even coherent states. Such states are useful, *e.g.*, for teleportation [18].

#### IV. PROBE FIELD PHOTO-DETECTION

So far we have assumed a coherent unitary evolution. Now, we turn to the situation where the probe light field is being detected. In several situations detection allows an additional control [13, 14]. Here we will see that the continuous photodetection over the probe field does not give any additional control over the single mode BEC phase.

We employ a continuous photodetection model [12, 17] characterized by a set of operations  $N_t(k)$ , such that,

$$\rho_k(t) = \frac{N_t(k) \rho(0)}{P(k, t)}, \quad (14)$$

where  $P(k, t) = \text{Tr}[N_t(k) \rho(0)]$  is the probability that  $k$  photocounts are observed during the time interval  $t$ . The operation

$$N_t(k) \rho = \int_0^t dt_k \int_0^{t_k} dt_{k-1} \int_0^{t_{k-1}} dt_{k-2} \dots \int_0^{t_1} dt_1 S_{t-t_k} J S_{t_k-t_{k-1}} \dots J S_{t_1} \rho \quad (15)$$

accounts for all possible one-count process, with  $J\rho = \gamma a \rho a^\dagger$  (where  $\gamma$  is the detector counting rate) followed by the non-unitary evolution between consecutive counts  $S_t \rho = e^{Yt} \rho e^{Y^\dagger t}$ , with  $Y = -i\frac{H}{\hbar} - \frac{\gamma}{2} a^\dagger a$ . After  $k$ -counts on the probe field, the conditioned joint state becomes

$$\rho_k(t) = \frac{1}{P(k, t) k!} \sum_{m, n} \mathcal{C}_{m, n}^{(k)}(t) |m\rangle \langle n| \otimes |\beta_m(t)\rangle \langle \beta_n(t)|, \quad (16)$$

where  $\mathcal{C}_{m, n}^{(k)}(t) = e^{-|\alpha|^2} \frac{\alpha^m \alpha^{*n}}{\sqrt{m!n!}} [\mathcal{F}_{m, n}]^k e^{\Phi_m(t) + \Phi_n^*(t)}$ , with

$$\mathcal{F}_{m, n}(t) = \gamma \left\{ -\frac{\Lambda_m \Lambda_n^*}{\Gamma_m + \Gamma_n^*} \left[ e^{-(\Gamma_m + \Gamma_n^*)t} - 1 \right] + G_m G_n^* t \right\} + i\gamma \left\{ \frac{G_m \Lambda_n^*}{\Gamma_n^*} f_n^*(t) - \frac{G_n^* \Lambda_m}{\Gamma_m} f_m(t) \right\}, \quad (17)$$

where  $f_m(t) \equiv (e^{-\Gamma_m t} - 1)$  and

$$\begin{aligned} \Phi_m(t) = & -\frac{1}{2} \left( |\beta|^2 - |\beta_m(t)|^2 \right) + iG_m \Lambda_m f_m(t) \\ & - |G_m|^2 \Gamma_m^* t - i \left[ (\tilde{\omega}_0 + \frac{|g_2|^2}{\Delta}) m + \kappa m(m-1) \right] t. \end{aligned} \quad (18)$$

In Eqs. (17) and (18),  $G_m = \frac{g_1 \tilde{g}_2}{\Delta \Gamma_m} m$ ,  $\Lambda_m = \beta + iG_m$ ,  $\Gamma_m = i\frac{|g_1|^2}{\Delta} m + \frac{\gamma}{2}$ , and  $\beta_m(t) = \Lambda_m e^{-\Gamma_m t} - iG_m$ . Besides  $P(k, t)$  is given by

$$P(k, t) = \frac{e^{-|\alpha|^2}}{k!} \sum_m \frac{(|\alpha|^2)^m}{m!} [\mathcal{F}_{m, m}(t)]^k e^{-\mathcal{F}_{m, m}(t)}. \quad (19)$$

In Fig. 3 we plot the phase variance and the corresponding probability of occurrence for (a)  $k = 0$  and (b)  $k = 1$ , respectively, for the same parameters found in Fig. 2(f), and with  $\gamma = 2 \times 10^{-2} \kappa$ . We see in Fig. 3(a) that the no-counting does not affect the revival of the state, at the typical time scale for a no-count event (Fig. 3(d)). We only see some change after a few collapse and revivals. At the revival time scale the probability that  $k = 1$  counting occurs is large enough so that the chance to get a revival of the BEC phase is severely compromised as we see in Fig. 3(b). In fact the same occurs for any  $k \neq 0$ . Therefore the phase evolution given by the postselected state is very sensitive, and whenever a photodetection event occurs the whole atom-light system state is so affected that there is no chance for a revival of the initial BEC phase. This contrasts to other situations where the detection process helps to define a phase. Therefore no further control is achieved, other than the one by the detuning of the optical pump frequency. For completeness, in Fig. 3(c) we plot the phase variance for unconditioned (pre-selected) state of the joint BEC-light system under the effect of counting,

$$\begin{aligned} \rho(t) &= \sum_k P(k, t) \rho_k(t) \\ &= \sum_{m,n} C_{m,n}(t) |\beta_m(t)\rangle \langle \beta_n(t)| \otimes |m\rangle \langle n|, \end{aligned} \quad (20)$$

with  $C_{m,n}(t) = e^{-|\alpha|^2} \frac{\alpha^m \alpha^{*n}}{\sqrt{m!n!}} e^{\Phi_m(t) + \Phi_n^*(t) + \mathcal{F}_{m,n}(t)}$ . This is the situation when one is absolutely ignorant about the counting events, and therefore the best one can do is to assume a convex sum of all conditioned states  $\rho_k(t)$  occurring with probability  $P(k, t)$ . Effectively the evolved state (20) is equivalent to the situation where the light field is under the action of an amplitude damping channel, due to contact with a zero-temperature reservoir. Therefore the pre-selected state reflects the joint system incoherent evolution.

We see in Fig 3(c) that there might be a chance of a partial or complete revival under damping depending on  $\gamma$ . The dependence on the counting rate is better seen in Fig. 4 where the phase variance value at the revival times is plotted against the variation of  $\gamma$  for the pre-selected state. We can see that above  $\gamma/2\kappa = 6 \times 10^{-4}$  ( $\gamma/2\kappa = 1.2 \times 10^{-3}$ ) there is not a single phase revival, inside a tolerance of 10% (20%). All the following revivals can be tracked in a similar manner and obviously will be more sensitive to the variation of  $\gamma$ . We remark that the observed effect for one counting event ( $k = 1$ ) would be similarly observed had we taken an arbitrary counting event, given by the state

$$\tilde{\rho}(t) = \frac{1}{1 - P(0, t)} [\rho(t) - P(0, t) \rho_0(t)], \quad (21)$$

occurring with probability  $1 - P(0, t)$ , with no further observed advantage than the one considered in Fig. 3.

## V. CONCLUSION

In conclusion, the observed collapse and revival of the macroscopic matter wave field of a BEC interacting with a

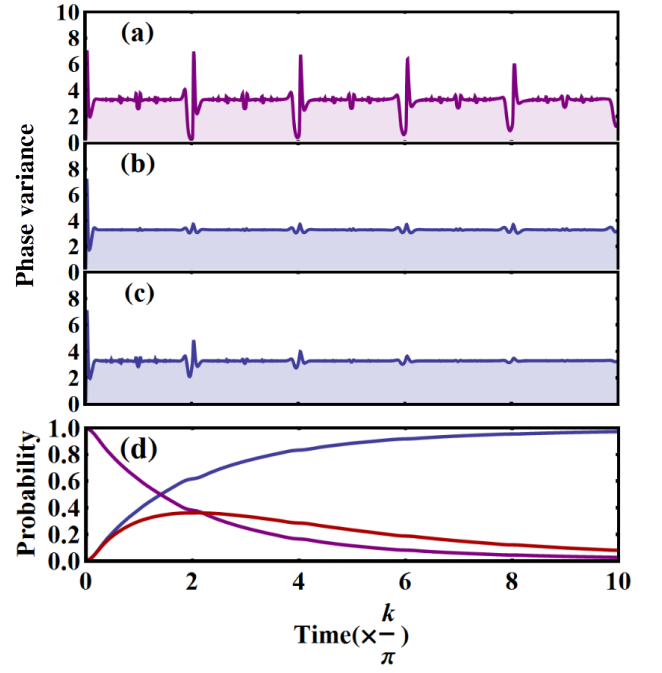


FIG. 3. The revival of the phase is severely affected by the counting process as depicted by the phase variance for the BEC with the photo-counting process at a  $\gamma = 2 \times 10^{-2} \kappa$  counting rate. Other parameters are similar to the ones in Fig. 2(f). (a) Variance for a no-count event, and (b) the variance for a single count event. (c) Variance for the pre-selected state (20). (d)  $P(0, t)$  (purple line),  $P(1, t)$  (red line), and probability of an arbitrary counting event,  $[1 - P(0, t)]$  (blue line).

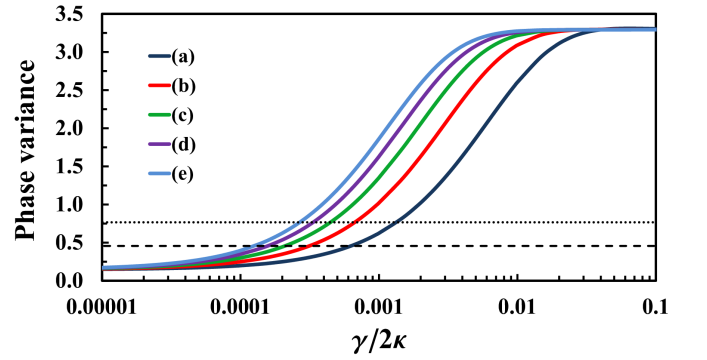


FIG. 4. Phase variance at the revival time calculated with the pre-selected state (20) as a function of the detector rate  $\gamma$  for the case shown in Fig. 2(f). (a)-(e) correspond to the first-five revivals, respectively. The dashed and dotted lines represent tolerance of 10% and 20% in the revival, respectively.

one-mode quantized optical probe, is thoroughly dependent on the atom-light interaction parameters. This dependence allows some degree of optical control on the atomic collapse and revival times by adjusting the coupling between the atoms and the optical field, through the variation of frequency detuning  $\Delta$ . In addition, we analyze the effects of dissipation and photodetection over the optical field upon the BEC phase dynamics to check whether an additional control would be possible.

We show that in contrast with other situations, such as the one present in double-well condensates [13], the phase revival for a single mode BEC is very sensitive to detection and never occurs for  $k \neq 0$  counting events on the optical probe. The overall effect of the detection is to induce a phase damping in the condensate. Therefore for the single mode BEC interacting with a quantum optical probe, the only possible control of phase collapse and revival is through the frequency detuning.

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### Appendix: Derivation of Equations (7), (8) and (9)

To obtain the state given by Eq. (7), we apply the propagator  $U(t) = \exp(-\frac{i}{\hbar}Ht)$  into the initial state given by Eq.  $|\psi(0)\rangle = |\alpha\rangle \otimes |\beta\rangle$  with  $H$  given by (6). Since the first two terms in the Hamiltonian (6) commute with the remaining ones, we can write the propagator as follows

$$U(t) = e^{-i[\omega_A n_0 + \kappa n_0(n_0-1)]t} e^{-i(Fa + F^* a^\dagger + \xi a^\dagger a)n_0 t}, \quad (\text{A.1})$$

where  $\omega_A = \tilde{\omega}_0 + \frac{|\tilde{g}_2|^2}{\Delta}$ ,  $n_0 = c_0^\dagger c_0$ ,  $F = \frac{g_1 \tilde{g}_2^*}{\Delta}$  and  $\xi = \frac{|g_1|^2}{\Delta}$ . By expanding the initial atomic state in the Fock basis, the application of the propagator over the global initial state is given by

$$U(t) |\psi(0)\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_m \frac{\alpha^2}{\sqrt{m!}} e^{-i[\omega_A m + \kappa m(m-1)]t} |m\rangle \otimes e^{-i(Fa + F^* a^\dagger + \xi a^\dagger a)mt} |\beta\rangle. \quad (\text{A.2})$$

To solve  $e^{-i[F_m a + F_m^* a^\dagger + \xi_m a^\dagger a]t} |\beta\rangle$ , where  $F_m = \frac{g_1 \tilde{g}_2^*}{\Delta} m$  and  $\xi_m = \frac{|g_1|^2}{\Delta} m$ , we employ the normal ordering method for solving Schrödinger equation [19]. The generator of the evolution is the Hamiltonian of a Driven Harmonic Oscillator:  $H_{DHO} = \hbar \xi_m a^\dagger a + \hbar F_m a + \hbar F_m^* a^\dagger$ . The Schrödinger equation

$$i\hbar \frac{\partial |\beta(t)\rangle}{\partial t} = H_{DHO} |\beta(t)\rangle, \quad (\text{A.3})$$

has a solution given by  $|\beta(t)\rangle = U_{DHO}(t, t_0) |\beta(t_0)\rangle$  where  $|\beta(t_0)\rangle = |\beta\rangle$  and  $U_{DHO}$  also satisfies

$$i\hbar \frac{\partial U_{DHO}}{\partial t} = H_{DHO} U_{DHO}, \quad (\text{A.4})$$

subject to the initial condition  $U_{DHO}(t_0, t_0) = 1$ . In general, a Hamiltonian  $H(a, a^\dagger, t)$  in the normal order is given by  $H(a, a^\dagger, t) = \sum_{l,m} h_{l,m}(t) a^{\dagger l} a^m$ , where  $h_{l,m}(t)$  are

$c$ -number expansion coefficients. The propagator for this Hamiltonian will satisfy the following equation

$$i\hbar \frac{\partial U}{\partial t} = \sum_{l,m} h_{l,m}(t) a^{\dagger l} a^m U. \quad (\text{A.5})$$

Now, consider the theorem [?] that says if  $m$  is an integer and  $f(a, a^\dagger) = f^{(n)}(a, a^\dagger)$  (where the superscript denotes normal order), then  $a^m f(a, a^\dagger) = \mathcal{N} \left\{ \left( \beta + \frac{\partial}{\partial \beta^*} \right)^m \bar{f}^{(n)}(\beta, \beta^*) \right\} = \mathcal{N} \left\{ \langle \beta | a^m f(a, a^\dagger) | \beta \rangle \right\}$ , where  $\bar{f}^{(n)}(\beta, \beta^*)$  is an ordinary function of the complex variable  $\beta$ , and  $\mathcal{N}$  is an operator that transforms an ordinary function  $\bar{f}^{(n)}(\beta, \beta^*)$  to an operator function  $f^{(n)}(a, a^\dagger)$  by replacing  $\beta$  by  $a$  and  $\beta^*$  by  $a^\dagger$ . With the help of this theorem we can rewrite (A.5) as

$$i\hbar \frac{\partial U}{\partial t} = \sum_{l,m} h_{l,m}(t) a^{\dagger l} \mathcal{N} \left\{ \left( \beta + \frac{\partial}{\partial \beta^*} \right)^m \bar{U}^{(n)}(\beta, \beta^*, t) \right\}, \quad (\text{A.6})$$

where  $\bar{U}^{(n)}(\beta, \beta^*, t) = \langle \beta | U(\beta, \beta^*, t) | \beta \rangle$ . If we take diagonal coherent state matrix elements of both sides of (A.6) we obtain the following  $c$ -number equation

$$i\hbar \frac{\partial \bar{U}^{(n)}}{\partial t} = \sum_{l,m} h_{l,m}(t) \beta^{*l} \left( \beta + \frac{\partial}{\partial \beta^*} \right)^m \bar{U}^{(n)}, \quad (\text{A.7})$$

since the right hand side is in normal order. Solving (A.7), we obtain  $|\beta(t)\rangle$  by  $|\beta(t)\rangle = \mathcal{N} \{ U^{(n)}(\beta, \beta^*, t) \} |\beta(t_0)\rangle$ . In our particular case, with the generator  $H_{DHO}$ , we have

$$i\hbar \frac{\partial \bar{U}^{(n)}}{\partial t} = \hbar \xi_m \beta^* \left( \beta + \frac{\partial}{\partial \beta^*} \right) \bar{U}^{(n)} + \hbar \left[ F_m \left( \beta + \frac{\partial}{\partial \beta^*} \right) + F_m^* \beta^* \right] \bar{U}^{(n)}. \quad (\text{A.8})$$

If  $\bar{U}^{(n)} = e^{G(\beta, \beta^*, t)}$  where  $G(\beta, \beta^*, t) = A(t) + B(t)\beta + C(t)\beta^* + D(t)\beta^*\beta$ , then (A.8) becomes  $i \left[ \frac{dA}{dt} + \frac{dB}{dt}\beta + \frac{dC}{dt}\beta^* + \frac{dD}{dt}\beta^*\beta \right] = \xi_m \beta^* \beta + \xi_m \beta^* (C + D\beta) + F_m \beta + F_m^* \beta^* + F_m (C + D\beta)$ , which can be separated in the following set of equations

$$i \frac{dD}{dt} = \xi_m (D + 1), \quad (\text{A.9})$$

$$i \frac{dB}{dt} = F_m (D + 1), \quad (\text{A.10})$$

$$i \frac{dC}{dt} = \xi_m C + F_m^*, \quad (\text{A.11})$$

$$i \frac{dA}{dt} = F_m C, \quad (\text{A.12})$$

and whose solutions are given by  $D(t) = e^{-i\xi_m t} - 1$ ,  $B(t) = \frac{F_m}{\xi_m} (e^{-i\xi_m t} - 1)$ ,  $C(t) = \frac{F_m^*}{\xi_m} (e^{-i\xi_m t} - 1)$  and  $A(t) = \frac{|F_m|^2}{\xi_m^2} (e^{-i\xi_m t} - 1) + i \frac{|F_m|^2}{\xi_m} t$ . Since

$$\begin{aligned} |\beta(t)\rangle &= U(t) |\beta\rangle = \mathcal{N} \left\{ e^{A+B\beta+C\beta^*+D\beta^*\beta} \right\} |\beta\rangle \\ &= e^{A(t)} e^{C(t)a^\dagger} \mathcal{N} \left\{ e^{D(t)\beta^*\beta} \right\} e^{B(t)\beta} |\beta\rangle, \quad (\text{A.13}) \end{aligned}$$

and  $f(a) |\beta\rangle = f(\beta) |\beta\rangle$ , then

$$|\beta(t)\rangle = e^{A(t)+B(t)\beta} e^{C(t)a^\dagger} e^{D(t)\beta a^\dagger} |\beta\rangle. \quad (\text{A.14})$$

By identifying  $|\beta\rangle = e^{-\frac{|\beta|^2}{2}} e^{\beta a^\dagger} |0\rangle$ , then we are able to recognise that

$$|\beta(t)\rangle = e^{A(t)+B(t)\beta-\frac{|\beta|^2}{2}} e^{\{[1+D(t)]\beta+C(t)\}a^\dagger} |0\rangle. \quad (\text{A.15})$$

If we substitute the  $A(t)$ ,  $B(t)$ ,  $C(t)$  and  $D(t)$ ,  $\xi_m$ , and  $F_m$  in (A.15) we recover the Equations (7), (8) and (9).

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- [1] M. Lewenstein and L. You, Phys. Rev. Lett. **77**, 3489 (1996).
  - [2] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
  - [3] K. B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
  - [4] M. Greiner, O. Mandel, T. W. Hansch, and I. Bloch, Nature **419**, 51 (2002).
  - [5] M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge University Press, 1997) p. 630.
  - [6] J. Lozada-Vera, V. S. Bagnato, and M. C. de Oliveira, New Journal of Physics **15**, 113012 (2013).
  - [7] C. Gross, T. Zibold, E. Nicklas, J. Estève, and M. K. Oberthaler, Nature **464**, 1165 (2010).
  - [8] I. B. Mekhov and H. Ritsch, Journal of Physics B: Atomic, Molecular and Optical Physics **45**, 102001 (2012).
  - [9] M. G. Moore and P. Meystre, Phys. Rev. A **59**, R1754 (1999).
  - [10] M. G. Moore, O. Zobay, and P. Meystre, Phys. Rev. A **60**, 1491 (1999).
  - [11] V. V. França and G. A. Pratavia, Phys. Rev. A **75**, 043604 (2007).
  - [12] G. A. Pratavia and M. C. de Oliveira, Phys. Rev. A **70**, 011602 (2004).
  - [13] M. Saba, T. A. Pasquini, C. Sanner, Y. Shin, W. Ketterle, and D. E. Pritchard, Science **307**, 1945 (2005), <http://science.sciencemag.org/content/307/5717/1945.full.pdf>.
  - [14] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Buchler, and P. Zoller, Nature Physics **4**, 878 (2008).
  - [15] V. V. França and G. A. Pratavia, Phys. Rev. A **75**, 043604 (2007).
  - [16] C. J. Pethick and H. Smith, Bose - Einstein Condensation in Dilute Gases, 2nd ed. (Cambridge Express, 2008).
  - [17] D. F. Walls and G. J. Milburn, Quantum Optics (Springer, 2008).
  - [18] M. C. de Oliveira, Phys. Rev. A **67**, 022307 (2003).
  - [19] W. H. Louisel, Quantum statistical properties of radiation (A Wiley-Interscience publication, 1973) p. 528.